OTPORNOST MATERIJALA

VEŽBE BR. 6

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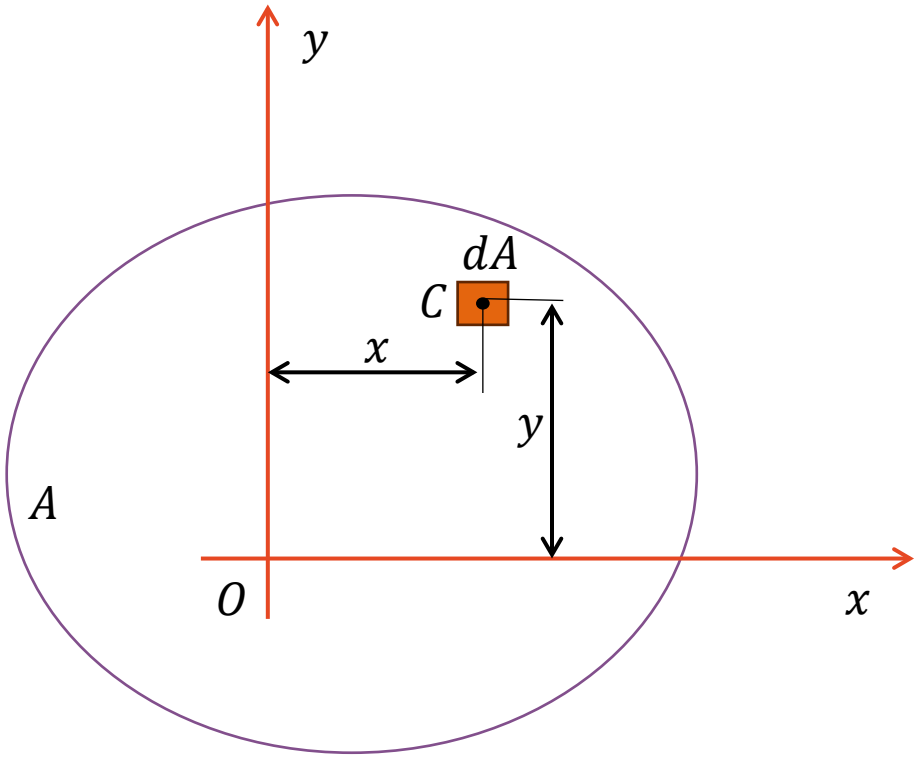
MOMENT INERCIJE

MOMENT INERCIJE RAVNIH POVRŠINA

AKSIJALNI MOMENT INERCIJE

Moment površine drugog reda jednak je zbiru proizvoda elementarne površine i kvadrata rastojanja od njenog težišta do odgovarajuće ose.

$$I_x = \int_A y^2 dA \qquad I_y = \int_A x^2 dA \qquad [cm^4]$$

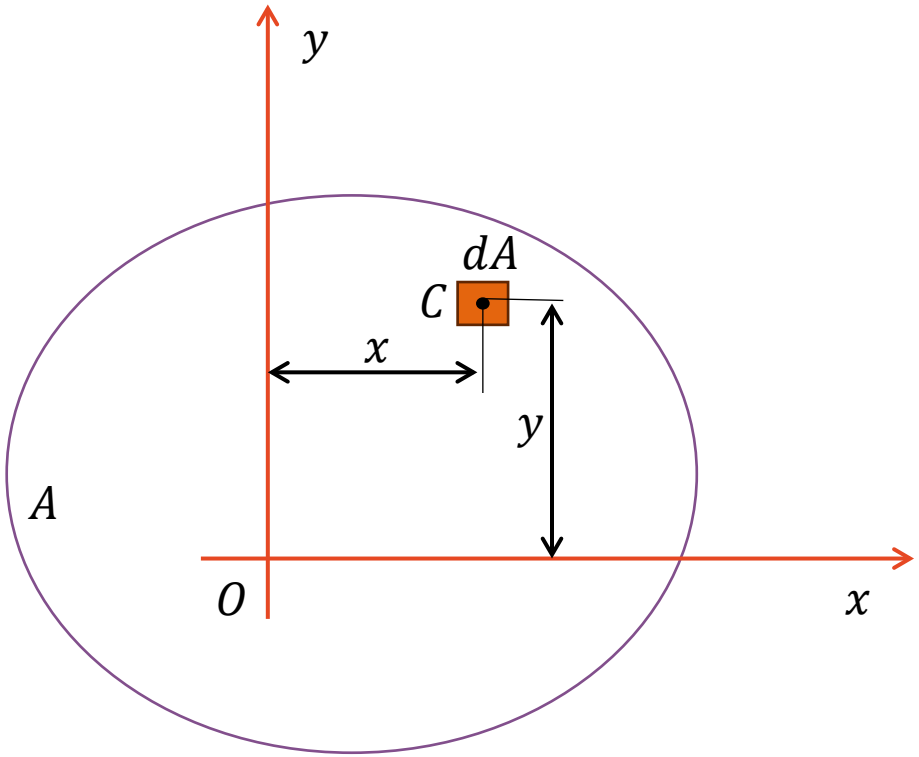


MOMENT INERCIJE RAVNIH POVRŠINA

POLARNI MOMENT INERCIJE

Za neku tačku (pol) jednak je zbiru proizvoda elementarne površine i kvadrata rastojanja od njenog težišta do te tačke (pola).

$$I_o = \int_A r^2 dA \qquad [cm^4]$$



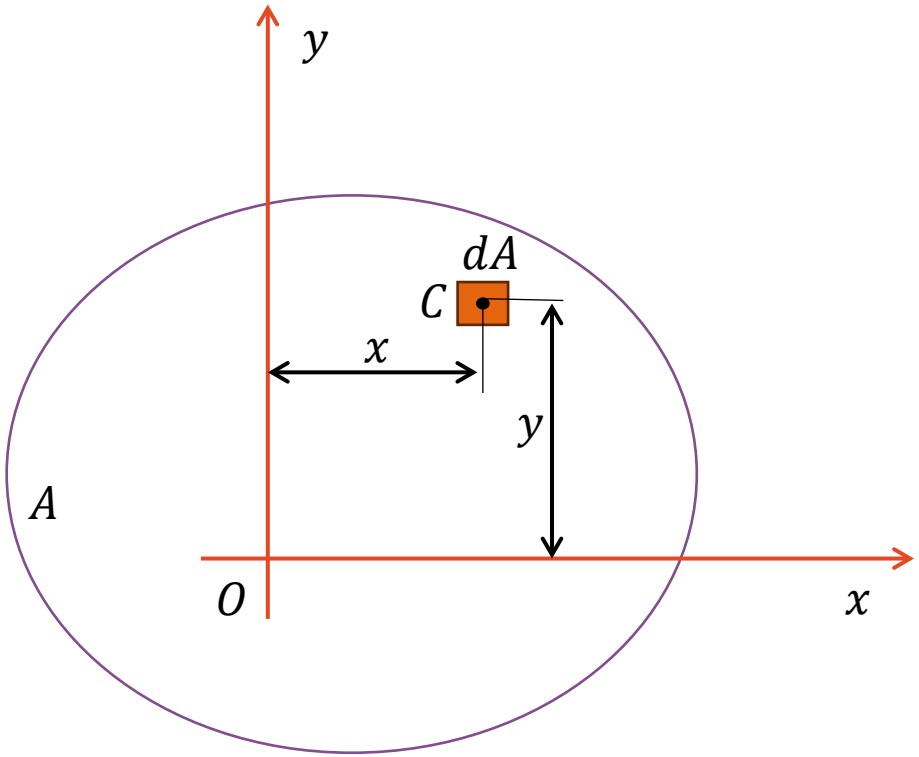
MOMENT INERCIJE RAVNIH POVRŠINA

OTPORNI MOMENT INERCIJE

$$W_O = \frac{I_O}{R}$$

$$R = \frac{d}{2} \; - polupre\check{c}nik$$

$$W_O = \frac{2I_O}{d}$$



MOMENT INERCIJE RAVNIH POVRŠINA

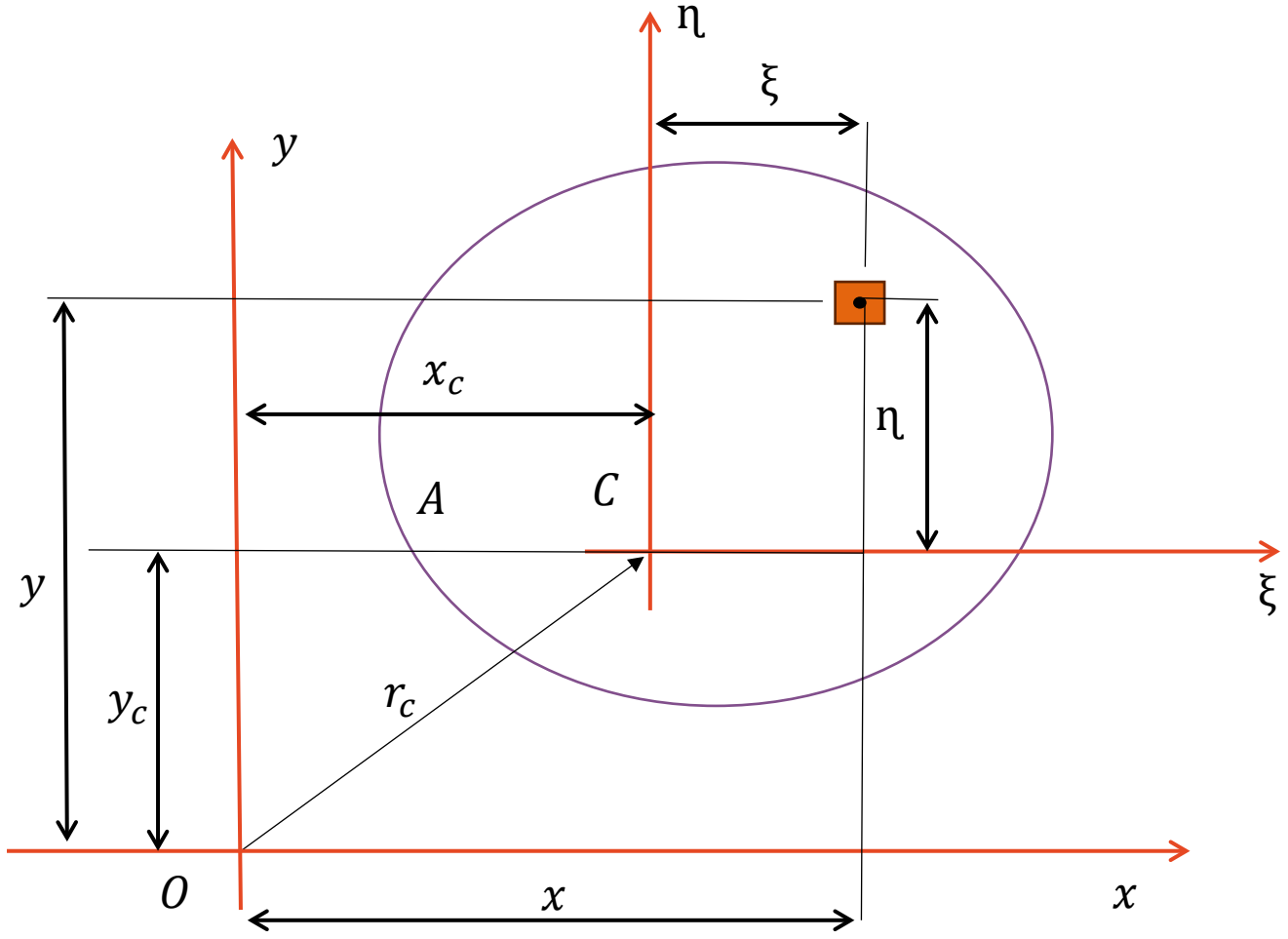
SOPSTVENI MOMENTI INERCIJE

$I_\xi \quad I_\eta$

AKSIJALNI MOMENT INERCIJE

$$I_X = I_\xi + Ay_c^2$$

$$I_y = I_\eta + Ax_c^2$$



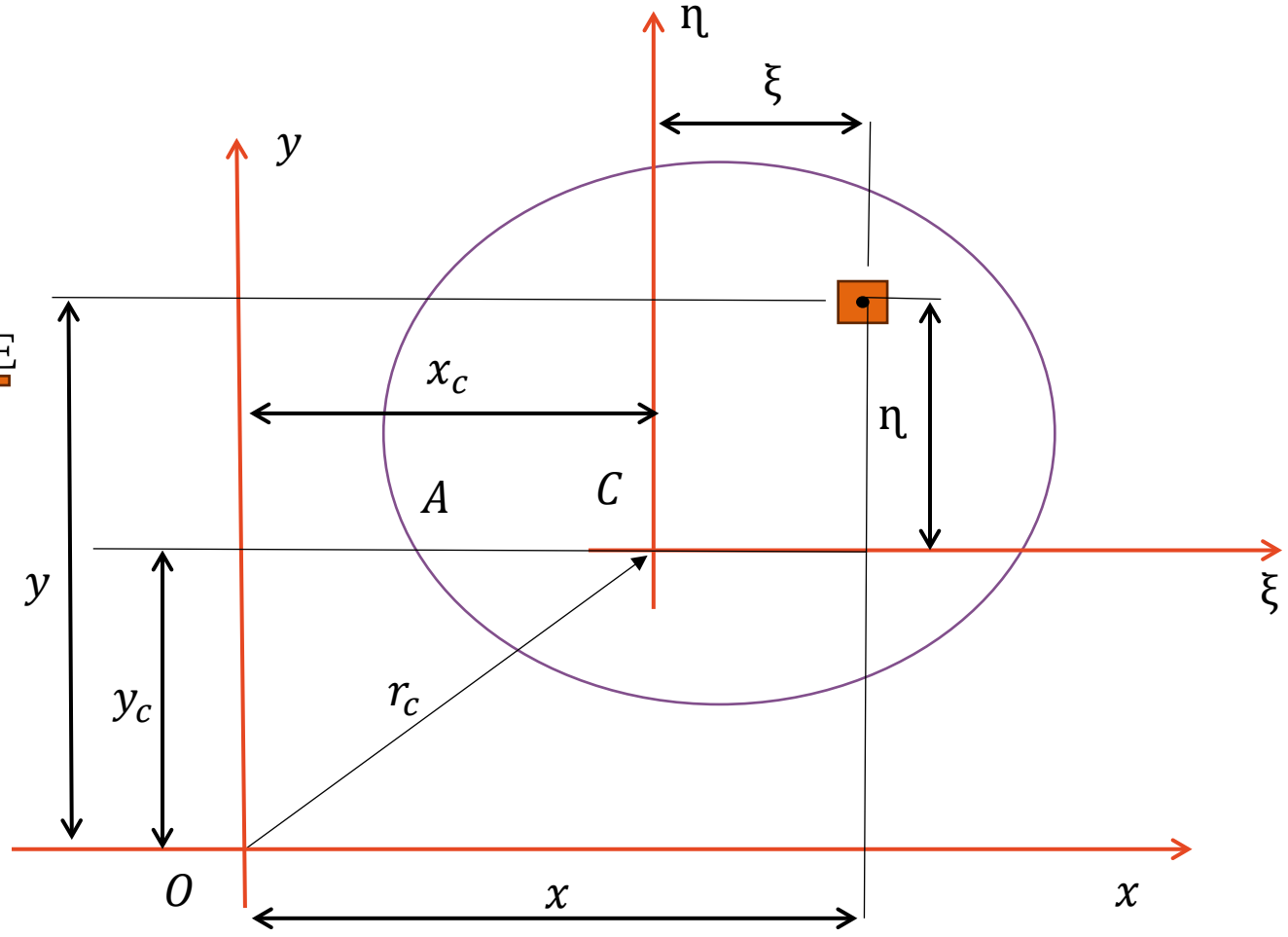
MOMENT INERCIJE RAVNIH POVRŠINA

POLARNI MOMENT INERCIJE

$$I_O = I_C + Ar_c^2$$

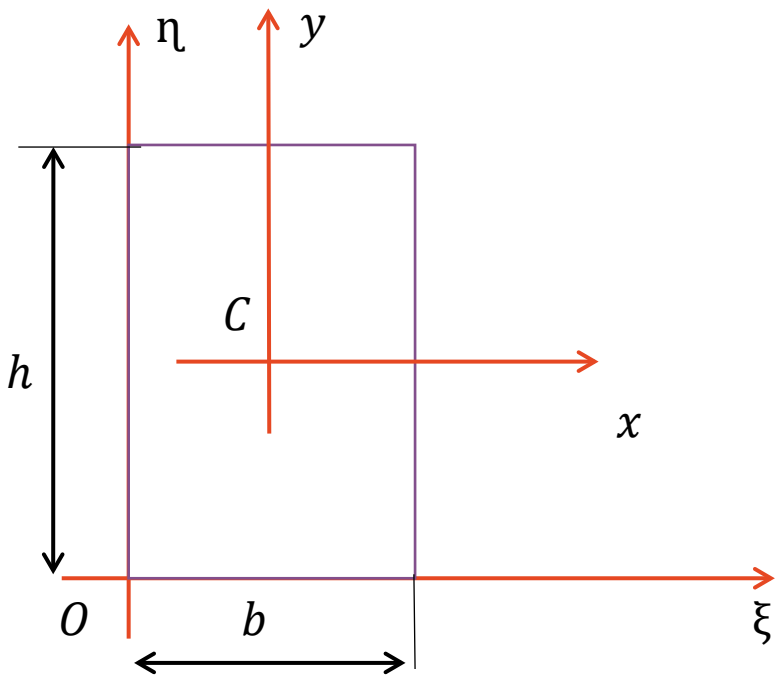
CENTRIFUGALNI MOMENT INERCIJE

$$I_{xy} = I_{\xi\eta} + Ax_c y_c$$



ZADATAK 1.

- Za pravougaonik dimenzija bxh=4x6cm, odrediti:
  - A) Sopstvene momente inercije za ose X i Y
  - B) Aksijalne momente inercije za ose ξ i η
  - C) Polarne momente inercije za tačke C i O.
  - D) Otporne momente inercije za ose X i Y.
  - E) Centrifugalni moment inercije za ose (x,y)



ZADATAK 1.

A) Sopstvene momente inercije za ose X i Y

$$I_x = \frac{bh^3}{12} = \frac{4 * 6^3}{12} = 72\ cm^4$$

$$I_y = \frac{hb^3}{12} = \frac{6 * 4^3}{12} = 32\ cm^4$$

B) Aksijalne momente inercije za ose ξ i η

$$I_\xi = I_x + Ay_c^2$$

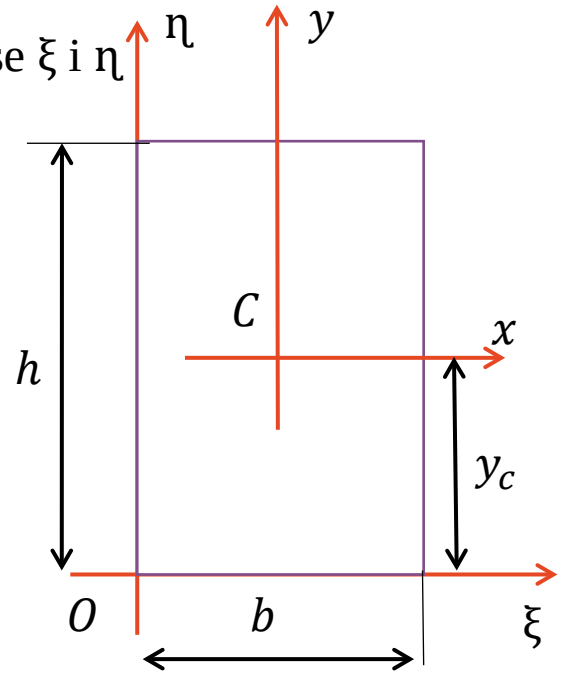
$$I_\xi = 72 + 4 * 6 * 3^2$$

$$I_\xi = 288\ cm^4$$

$$I_\eta = I_y + Ax_c^2$$

$$I_\eta = 32 + 4 * 6 * 2^2$$

$$I_\eta = 128\ cm^4$$



ZADATAK 1.

C) Polarne momente inercije za tačke C i O

$$I_C = I_X + I_y$$

$$I_C = 72 + 32$$

$$I_C = 104\ cm^4$$

$$I_O = I_\xi + I_\eta$$

$$I_O = 288 + 128$$

$$I_O = 416\ cm^4$$

D) Otporne momente inercije za ose X i Y

$$W_x = \frac{I_x}{e_x} \qquad e_x = \frac{h}{2}$$

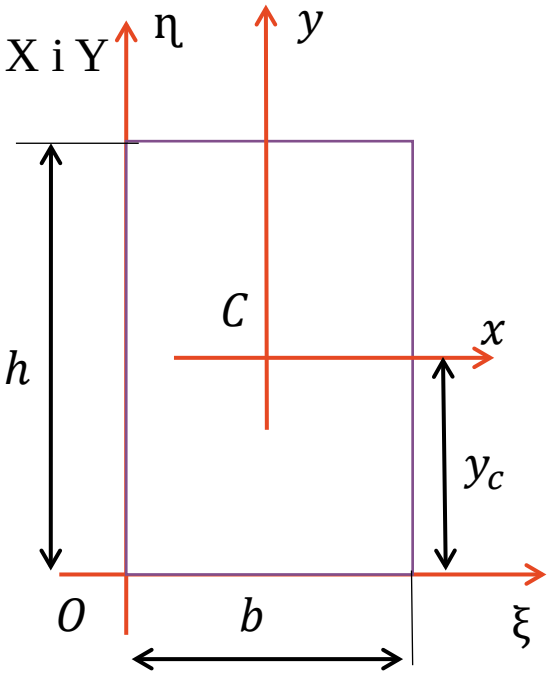
$$W_x = \frac{72}{3}$$

$$W_x = 24\ cm^3$$

$$W_y = \frac{I_y}{e_y} \qquad e_y = \frac{b}{2}$$

$$W_y = \frac{32}{2}$$

$$W_y = 16\ cm^3$$



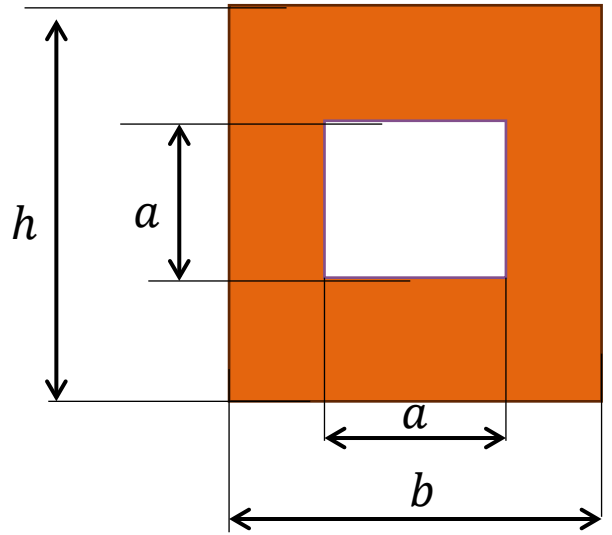
ZADATAK 1.

E) Centrifugalne momente inercije za ose (x,y)

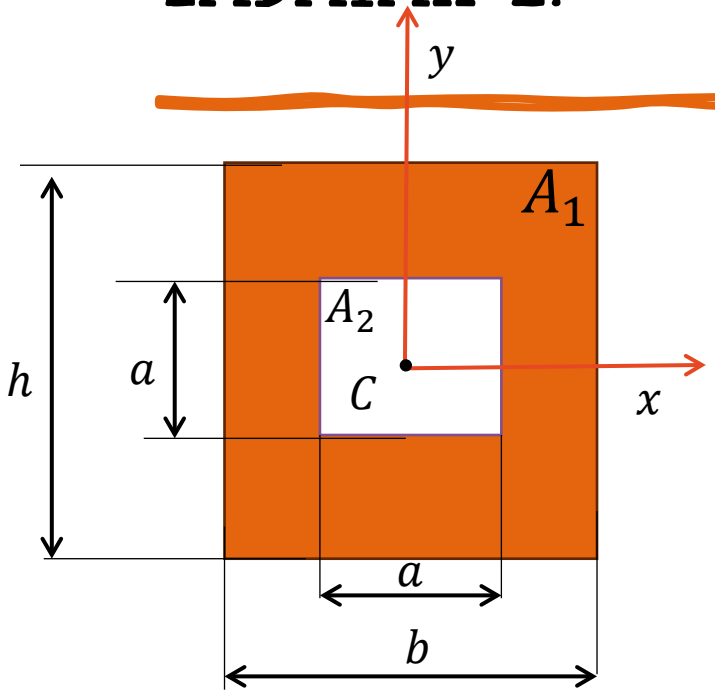
$$I_{xy} = 0 \quad - x \; i \; y \; ose \; su \; ose \; simetrije$$

ZADATAK 2.

- Izračunati moment inercije za težišne ose i težište površine poprečnog preseka datog na slici, ako je $a = 2cm, b = 4cm, h = 8cm$.



ZADATAK 2.



$$A_1 = b * h = 4 * 8 = 32\ cm^2$$

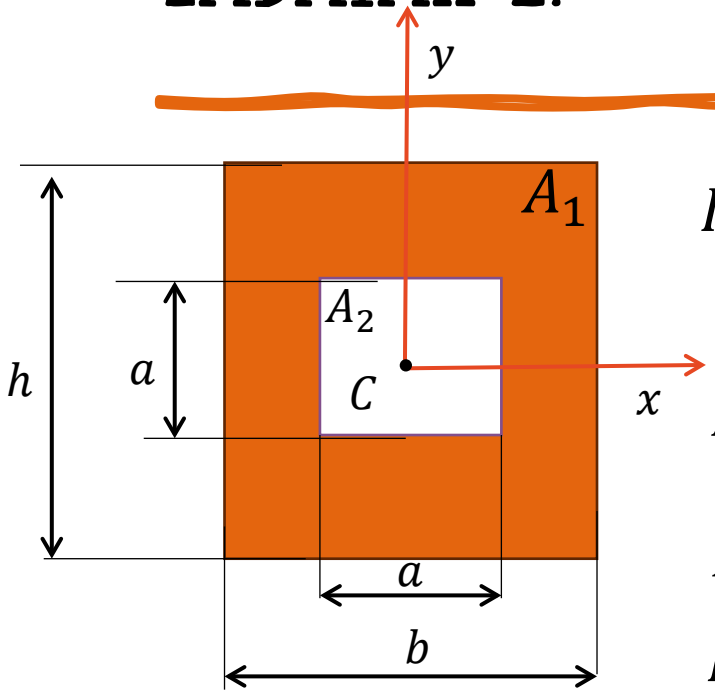
$$A_2 = a * a = 2 * 2 = 4\ cm^2$$

Moment inercije složene površine

$$I_x = I_{x1} - I_{x2}$$

$$I_y = I_{y1} - I_{y2}$$

ZADATAK 2.



$$I_{x1} = \frac{b * h^3}{12} = \frac{4 * 8^3}{12} = 170{,}67\ cm^4$$

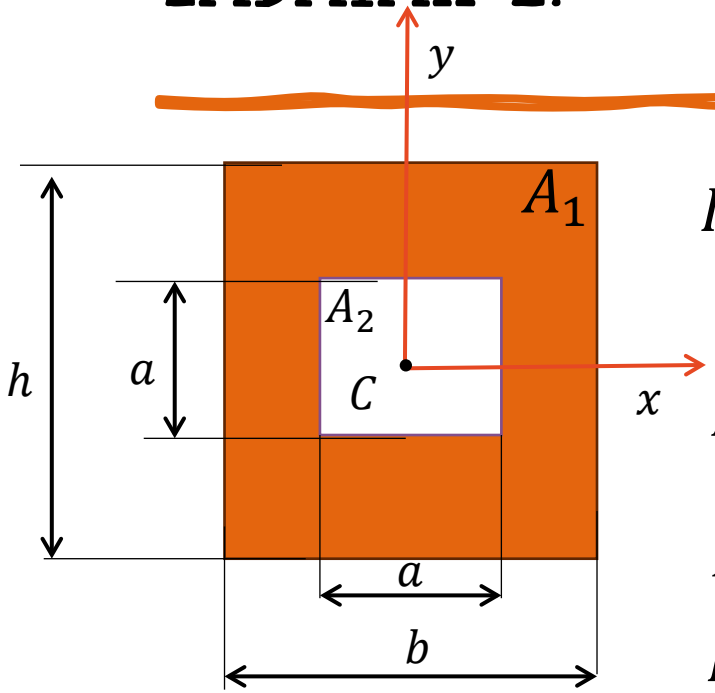
$$I_{x2} = \frac{a^4}{12} = \frac{2^4}{12} = 1{,}34\ cm^4$$

$$I_x = I_{x1} - I_{x2}$$

$$I_x = 170{,}67 - 1{,}34$$

$$I_x = 169{,}33\ cm^4$$

ZADATAK 2.



$$I_{y1} = \frac{h * b^3}{12} = \frac{8 * 4^3}{12} = 42{,}667\ cm^4$$

$$I_{y2} = \frac{a^4}{12} = \frac{2^4}{12} = 1{,}34\ cm^4$$

$$I_y = I_{y1} - I_{y2}$$

$$I_y = 42{,}667 - 1{,}34$$

$$I_y = 41{,}33\ cm^4$$

Polarni moment inercije

$$I_c = I_X + I_y$$

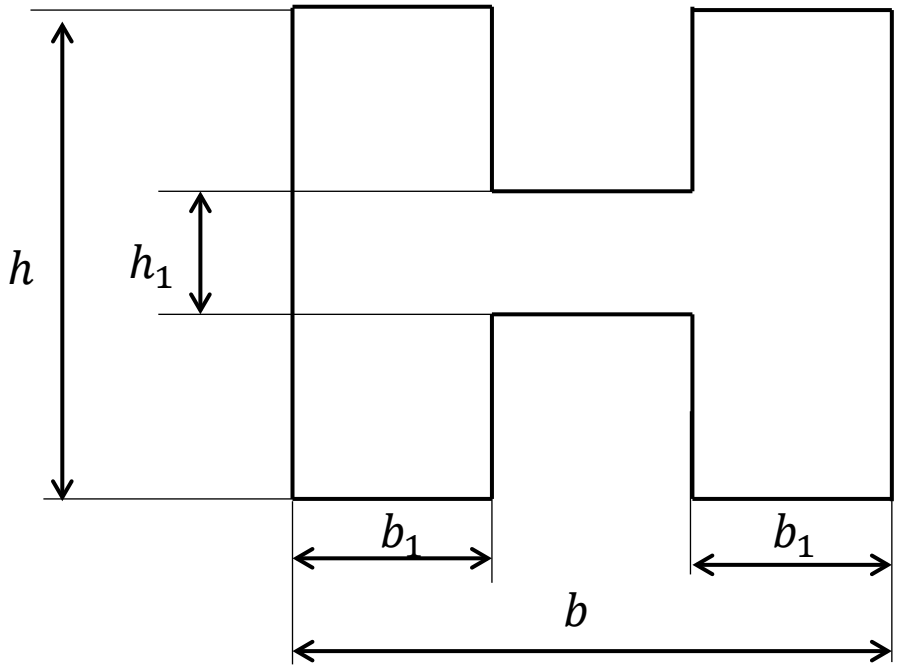
$$I_c = 210{,}66\ cm^4$$

Centrifugalni moment inercije

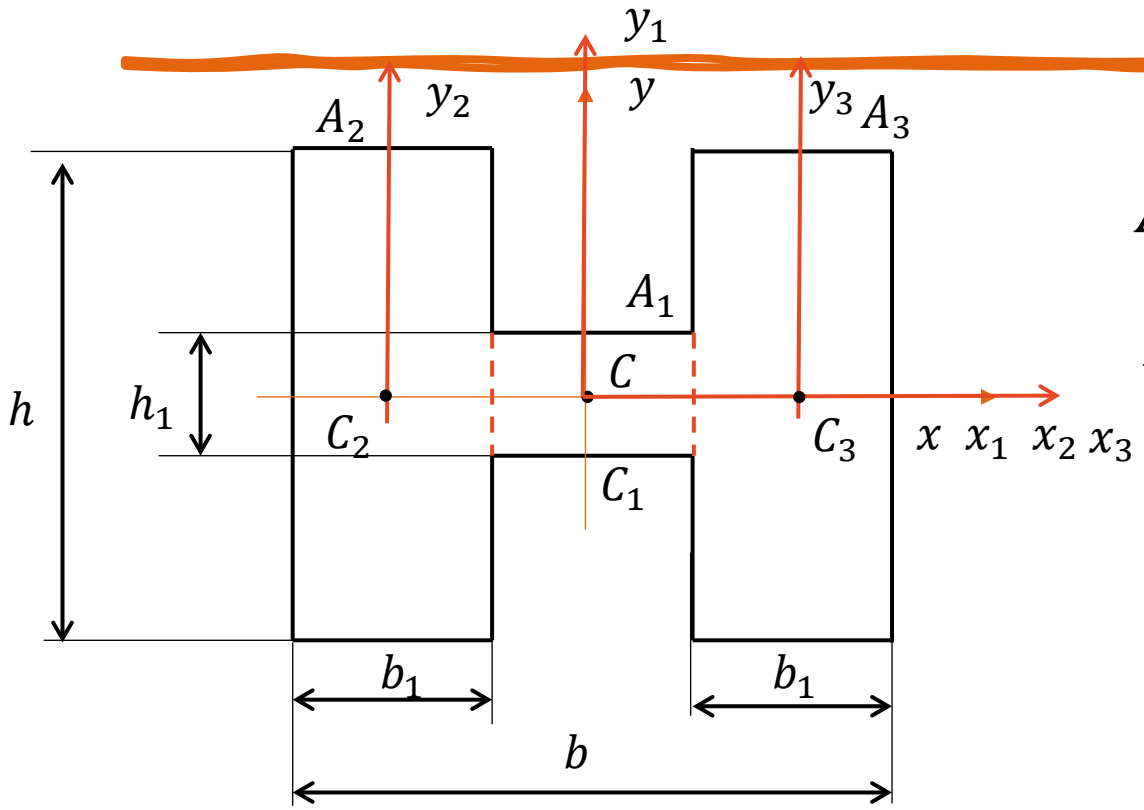
$$I_{xy} = 0$$

ZADATAK 3.

- Izračunati moment inercije za težišne ose i težište površine poprečnog preseka datog na slici, ako je $h_1 = 3cm, b_1 = 2cm, b = 10cm, h = 12cm$.



ZADATAK 3.



$$A_1 = (b - 2b_1) * h_1 = (10 - 4) * 3 = 18\ cm^2$$

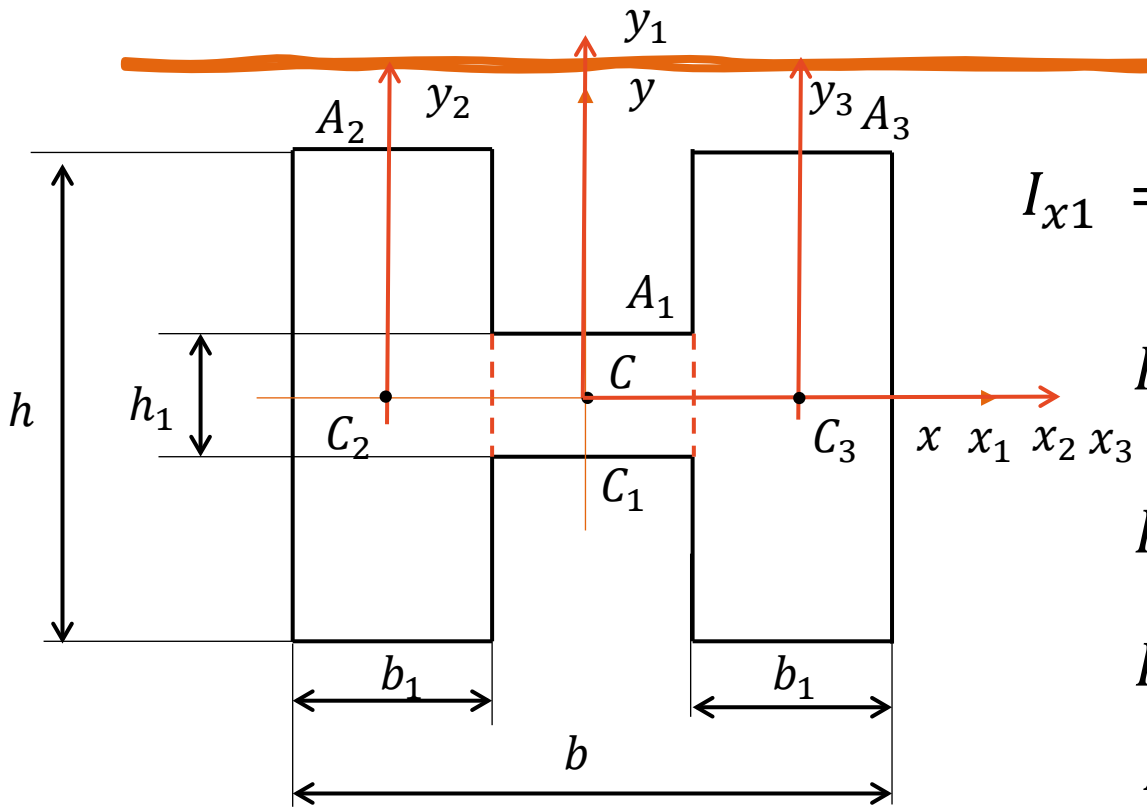
$$A_2 = A_3 = b_1 * h = 2 * 12 = 24\ cm^2$$

Moment inercije složene površine

$$I_x = I_{x1} + I_{x2} + I_{x3}$$

$$I_y = I_{y1} + I_{y2} + I_{y3}$$

ZADATAK 3.



$$I_{x1} = \frac{(b - 2b_1) * h_1^3}{12} = \frac{(10 - 2 * 2) * 3^3}{12} = 13.5\ cm^4$$

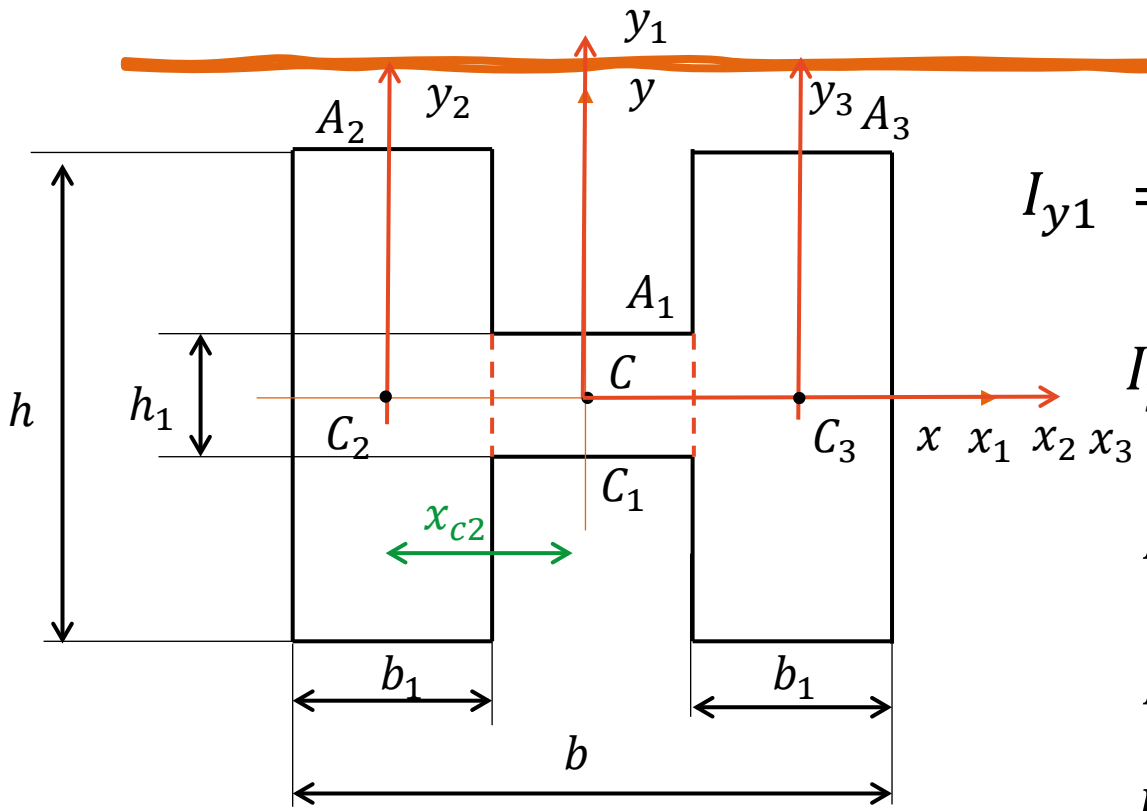
$$I_{x2} = I_{x3} = \frac{b_1 * h^3}{12} = \frac{2 * 12^3}{12} = 288\ cm^4$$

$$I_x = I_{x1} + I_{x2} + I_{x3}$$

$$I_x = 13{,}5 + 288 + 288$$

$$I_x = 589{,}5\ cm^4$$

ZADATAK 3.



$$I_{y1} = \frac{(b-2b_1)^3 * h_1}{12} = \frac{(10-2*2)^3 * 3}{12} = 54cm^4$$

$$I_{y2} = I_{y3} = I_{\text{ŋ}} + A_2 x_{c2}^2 = \frac{{b_1}^3 * h}{12} + A_2 * (\frac{b}{2} - \frac{b_1}{2})^2$$

$$I_{y2} = I_{y3} = \frac{2^3 * 12}{12} + 24 * (\frac{10}{2} - \frac{2}{2})^2$$

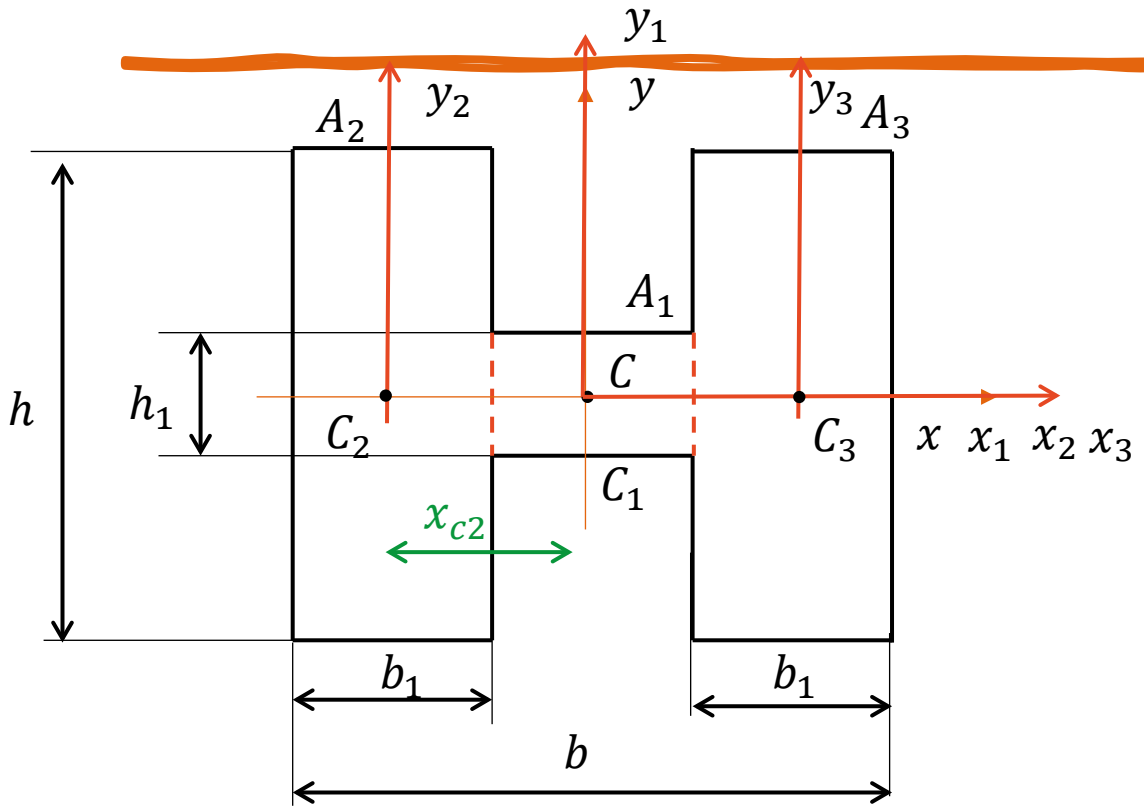
$$I_{y2} = I_{y3} = 392cm^4$$

$$I_{\text{y}} = I_{y1} + I_{y2} + I_{y3}$$

$$I_{\text{y}} = 54 + 392 + 392$$

$$I_{\text{y}} = 838\, cm^4$$

ZADATAK 3.



Polarni moment inercije

$$I_C = I_X + I_y$$

$$I_C = 589{,}5 + 838$$

$$I_C = 1427{,}5\ cm^4$$

Centrifugalni moment inercije

$$I_{XY} = 0 \quad - x \text{ i } y \text{ ose su ose simetrije}$$

HVALA NA PAŽNJI!